Participants, List of Titles, and Abstracts, Workshop 15w5049

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KINETIC MODELS OF GRANULAR AND OPTIMAL TRANSPORT I: GENERAL LOCAL EXISTENCE

Abstract: Granular materials consist of a large number of discrete solid particles (such as grains) which interact by nearly instantaneous collisions, much like in the classical model of a gas; but contrarily to ideal gas particle collisions, collisions of granular particle are inelastic, that is, characterized by a loss of kinetic energy. In this talk, we'll prove the local existence of weak solutions to a class of kinetic models of granular media. The existence proof relies on a splitting method (separating advection in position and interaction in velocity) where the spatially homogeneous equation is interpreted as the gradient flow of a convex interaction energy (in the velocity space) with respect to the Wasserstein distance.

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THE RADIATIVE TRANSFER EQUATION IN THE FORWARD-PEAKED REGIME

Abstract: In this lecture we present the radiative transfer equation in the forward-peaked regime in free space. We show instantaneous regularization of solution using hypo-elliptic techniques, convergence of the Henyey-Greenstein scattering models towards the peaked regime and time vanishing of solutions due to scattering. The analysis of the scattering operator is performed through elementary use of the stereographic projection, which renders a precise representation of the scattering mechanism in terms of a fractional Laplace-Beltrami operator on the sphere.

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ON THE ROTATION CURVES FOR AXIALLY SYMMETRIC DISK SOLUTIONS OF THE VLASOV-POISSON SYSTEM

Abstract: A large class of flat axially symmetric solutions to the Vlasov-Poisson system is constructed with the property that the corresponding rotation curves are approximately flat, slightly decreasing or slightly increasing. The rotation curves are compared with measurements from real galaxies and satisfactory agreement is obtained. These facts raise the question whether the observed rotation curves for disk galaxies may be explained without introducing dark matter. Furthermore, it is shown that for the ansatz we consider stars on circular orbits do not exist in the neighborhood of the boundary of the steady state.

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NONLINEAR ACOUSTIC WAVE PROPAGATION IN A RAREFIED GAS: NUMERICAL ANALYSIS BASED ON KINETIC AND FLUID EQUATIONS

Abstract: Nonlinear acoustic waves caused by an infinitely wide plate oscillating in its normal direction and propagating into a semi-infinite expanse of a rarefied gas are investigated numerically on the basis of a model Boltzmann equation and of the compressible Navier-Stokes equations with the correct temperature jump condition on the oscillating plate. The latter condition consists of the usual temperature jump proportional to the temperature gradient normal to the plate and the jump proportional to the velocity gradient normal to it. The long-time behavior of the solution, including the attenuation of the waves and the creation of the acoustic stream, is obtained accurately.

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A KINETIC MODEL FOR AN AGENT BASED MARKET SIMULATION

Abstract. A kinetic model for a specific agent based simulation to generate the sales curves of successive generations of high-end computer chips is developed. The resulting continuum market model consists of transport equations in two variables, representing the availability of money and the desire to buy a new chip. In lieu of typical collision terms in the kinetic equations that discontinuously change the attributes of an agent, discontinuous changes are initiated via boundary conditions between sets of partial differential equations. A scaling analysis of the transport equations determines the different time scales that constitute the market forces, characterizing different sales scenarios. It is argued that the resulting model can be adjusted to generic markets of multi-generational technology products where the innovation time scale is an important driver of the market.

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POLYNOMIAL RATE OF CONVERGENCE FOR ERGODIC AVERAGES ALONG VECTOR FIELDS

Abstract: Von Neumann’s original proof of the ergodic theorem for one-parameter families of unitary operators relies on a delicate analysis of the spectral measure of the associated flow operator and the observation that over long times only functions that are invariant under the flow make a contribution to the ergodic integral. In this talk I shall show that for a specific class of generators - namely vector fields - the spectral measure is rather simple to understand. For some nicely behaved flows this allows us to obtain a uniform ergodic theorem (note that there's no spectral gap!) with a polynomial rate of convergence. The analysis is performed in both Sobolev and weighted-Sobolev spaces. These results are a first step towards a better understanding of long time behavior of flows governed by the Vlasov equation.

Partially joint work with Clement Mouhot

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PROPAGATION IN KINETIC REACTION-TRANSPORT EQUATIONS

Abstract: We study travelling wave solutions of a kinetic reaction transport equation. The model describes particles moving according to a velocity-jump process, and proliferating thanks to a reaction term of monostable type. The boundedness of the velocity set appears to be a necessary and sufficient condition for the existence of positive travelling waves. The minimal speed of propagation of waves is obtained from an explicit dispersion relation. In case of an unbounded velocity set, we prove a superlinear spreading. It appears that the rate of spreading depends on the decay at infinity of the velocity distribution.

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VELOCITY-JUMP PROCESSES : LARGE DEVIATIONS AND ACCELERATION OF TRANSPORT-REACTION FRONTS

Abstract: I will present WKB asymptotics for a simple velocity-jump process involving free transport and reorientation at a constant rate following a Gaussian velocity distribution. I will highlight the particular scaling of large deviations, as opposed to the one for the diffusive limit. I will derive the corresponding nonlocal Hamilton-Jacobi verified by the action functional. I will present applications of this work to the accelaration of transport-reaction fronts, as opposed to constant speed of propagation of reaction-diffusion travelling waves.

This is joint work with Emeric Bouin (ENSL), Emmanuel Grenier (ENSL) and Grégoire Nadin (UPMC).

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CERCIGNANI'S CONJECTURE BETWEEN MULTIPLES OF THE EQUILIBRIUM

Abstract: The question of whether there exists a functional inequality that bounds the relative entropy by its production rate in the Boltzmann equation is known as Cercignani's conjecture. One of the reasons it is interesting is that it gives important information on the asymptotic behaviour of the equation. Unfortunately, it is known not to hold in general, even if one imposes quite strong conditions on the set of functions for which it is sought. We present a result that tests this on extremely strong conditions: we show that Cercignani's conjecture holds on the set S of all functions with fixed invariants (mass, energy and momentum) and which are bounded above and below by two fixed multiples of the equilibrium Maxwellian distribution (the one with the same invariants). We will also present some work in progress towards deducing some consequences on the behaviour of solutions to the space-homogeneous Boltzmann equation, the most important difficulty for this being that this set S is not known to be invariant by the flow of the equation.

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CLASSICAL AND QUANTUM MASTER EQUATIONS FOR N-PARTICLE SYSTEMS AND THEIR KINETIC LIMITS

Abstract: We present recent results as well as background on models for systems of N particles undergoing random binary collisions, focusing on propagation of chaos and the rate of convergence to equilibrium. These questions arise from the work of Mark Kac and his investigation into the probabilistic structure underlying the Boltzmann equation.

Recently, the quantum mechanical variation on Kac's question has begun to be investigated. There are novel difficulties due to the fact that in quantum mechanics, conditional probability is not always well defined. Nonetheless, a substantial quantum analogue of of the Kac program can be carried out, and this leads to an interesting and novel class of quantum kinetic equations.

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KINETIC MODELS OF GRANULAR AND OPTIMAL TRANSPORT II: ONE-DIMENSIONAL MODELS

Abstract: The aim of this talk is to show that for one-dimensional kinetic models of granular media, global results can be obtained for certain interaction kernels. For kernels whose second derivative is sub-quadratic near zero (which does not cover the quadratic case), we'll prove a global entropy bound (joint with M. Agueh and R. Illner). In the quadratic kernel case, there is a first integral of motion which enables us to reformulate the model as a gradient flow and then to obtain global existence (joint with M. Agueh).

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A BOLTZMANN MODEL FOR ROD ALIGNMENT AND SCHOOLING OF FISH

Abstract: We consider a Boltzmann model introduced by Bertin, Droz and Gregoire as a binary interaction model of the Vicsek alignment interaction. This model considers particles lying on the circle. Pairs of particles interact by trying to reach their mid-point (on the circle) up to some noise. We study the equilibria of this Boltzmann model and we rigorously show the existence of a pitchfork bifurcation when a parameter measuring the inverse of the noise intensity crosses a critical threshold. The analysisis carried over rigorously when there are only finitely many non-zero Fourier modes of the noise distribution. In this case, we can show that the critical exponent of the bifurcation is exactly 1/2. In the case of an infinite number of non-zero Fourier modes, a similar behavior can be formally obtained thanks to a method relying on integer partitions first proposed by Ben-Naim and Krapivsky. This is joint work with Eric Carlen, Pierre Degond and Bernt Wennberg.

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A LINEARLY TRANSFORMED PARTICLE METHOD FOR AGGREGATION EQUATION

Abstract: Particle methods are widely used for numerical resolution of transport equations. Conceptually simple and robust, classical smooth particle methods have the main drawback to often produce noisy solutions. The idea of the Linearly Transformed Particle method, developped by M. Campos-Pinto, is to transform the shape functions of particles in order to follow the local variation of the flot.
We adapt here this method for the numerical approximation of an aggregation equation with smooth or singular forces. For the smooth interaction forces, we provide convergence estimates in L^1 and L^{\infty} norms, and also convergence estimates in bounded Lipshitz distance for the measure valued solutions. For singular interaction potential, we establish the convergence of the error between the approximated and exact flows up to the existence time of the solutions in L^1 and L^p norm.

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ENTROPY METHODS AND SHARP FUNCTIONAL INEQUALITIES: NEW RESULTS

Abstract: Nonlinear diffusion flows and entropy methods provide very interesting tools for the study of sharp functional inequalities, the stability of the optimal functions and for improvements of the inequalities. The lecture will be devoted to a review of some results which unify some rigidity methods in nonlinear elliptic equations, the Bakry-Emery method and various qualitative features related with functional inequalities in probability theory, information theory and nonlinear analysis. The most striking result is a symmetry result for Caffarelli-Kohn-Nirenberg inequalities which has recently been obtained in collaboration with Maria J. Esteban and M. Loss, with applications to the spectral theory of Schrödinger operators.

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THE VLASOV-POISSON-BOLTZMANN SYSTEM AROUND A NONTRIVIAL PROFILE

Abstract: The talk is devoted to the study of the time-asymptotic stability of rarefaction waves for the Vlasov-Poisson-Boltzmann system in the whole space with slab symmetry. The large time profile of the electric potential can take distinct constant states at both far-fields. The rarefaction wave is constructed through the quasi neutral Euler equations which are the zero-order fluid dynamic approximation of the kinetic system. The key point in the proof is motivated by the study of the same problem for the viscous compressible fluid with the self-consistent electric field. The ratio of masses of two species particles plays a role in the construction and stability analysis of such nontrivial profile.

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ON THE CAUCHY THEORY FOR THE BOLTZMANN-NORDHEIM EQUATION FOR BOSONS

Abstract: One of the most influential equations in the kinetic theory of gases is the so-called Boltzmann equation, describing the time evolution of the probability density of a particle in a classical dilute gas. The irrefutable appearance of Quantum Mechanics, however, required a modification to this celebrated kinetic equation, resulting in the celebrated Boltzmann-Nordheim equation.

In our talk we will present a newly found local Cauchy Theory for the spatially homogeneous bosonic Boltzmann-Nordheim equation in any dimension $d \geq 3$ under mild restrictions on the collision kernel, extending previous studies that only dealt with the isotropic settings of the problem. Interestingly enough, the locality of this result is quite sharp due to the so-called Bose-Einstein condensation.

The methods used to achieve this theory are similar to those available for the classical Boltzmann equation, yet are entangled with L^∞ control that dominates the difference between the classical and quantum kinetic equation.

Time permitting we will discuss some details about the existence of a global solution to the equation.

This is a joint work with Marc Briant.

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HIGH ORDER SEMI-IMPLICIT SCHEMES FOR TIME DEPENDENT KINETIC EQUATIONS

Abstract: We consider a new formulation of implicit-explicit (IMEX) Runge-Kutta (R-K) methods for the numerical discretization of time dependent partial differential equations. The approach is based on identifying the (linear) dependence on the unknown of the system which generates the stiffness. Only the stiff dependence is treated implicitly, then making the whole method much simpler than fully implicit ones. This approach generalizes classical IMEX methods based on additive and partitioned R-K, and allows a novel application of semi-implicit schemes. We adopt several semi-implicit R-K methods up to order three. We illustrate the effectiveness of the new approach with many applications to reaction-diffusion, convection diffusion and nonlinear diffusion system of equations. Finally, we conclude by a stability analysis of the schemes for linear problems.

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FROM RADIATION IN CLOUDS TO FRACTIONAL DIFFUSION

Abstract: We investigate a kinetic equation that describes non-classical particle transport in media with correlated scattering centers. An example is water droplets in clouds. This equation is a time-dependent linear kinetic equation, whose initial values are given by a functional of its solution. We discuss the model's connection to particle billiards. An asymptotic analysis in the limit of small mean free paths reveals a fractional diffusion equation.

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ON THE KINETIC EQUATIONS OF A HARD SPHERE SYSTEM

Abstract: In the talk we consider a new approach to the description of the kinetic evolution of hard spheres within the framework of the marginal observables governed by the dual BBGKY hierarchy. The relations of the hierarchy of evolution equations for marginal observables and the nonlinear kinetic equations for states described in terms of a one-particle marginal distribution function are established [1].

Using suggested approach, the Boltzmann--Grad asymptotic behavior of a solution of the Cauchy problem of the dual BBGKY hierarchy with hard sphere collisions is constructed. The stated asymptotics is governed by the set of recurrence evolution equations, namely the dual Boltzmann hierarchy. For initial states specified in terms of a one-particle distribution function we prove that the mean value functional for the constructed limit additive-type marginal observables is equivalent to the mean value functional determined by a one-particle distribution function governed by the Boltzmann kinetic equation. In the general case of the limit $k > 1$-ary-type marginal observable the corresponding mean value functional are equivalent to the mean value functional determined by the k times product of a solution of the derived Boltzmann equation, i.e. in such a way we prove the property of the propagation of initial chaos [3].

One of the advantages of the developed approach to the derivation of the Boltzmann kinetic equation from underlying hard sphere dynamics consists in the possibility to construct the Boltzmann-type kinetic equation with initial correlations [2], for instance, correlations, characterizing the condensed states. Within the framework of this approach we also describe the propagation of initial correlations.

Moreover, using suggested approach, we derive the generalized Enskog equation [3]. The Boltzmann--Grad asymptotic behavior of its non-perturbative solution and the marginal functionals of states, which describe the creation of all possible correlations of particles with hard sphere collisions in terms of a one-particle distribution function, are established.

The obtained results we extend on systems of hard spheres with inelastic collisions [4]. In particular, we establish that in a one-dimensional space the kinetic evolution of a granular gas is governed by the certain generalization of the known Boltzmann equation for a one-dimensional granular gas and we construct the functionals of a solution of the derived kinetic equation that describe how the initial chaos propagates in one-dimensional granular gases.

1. Gerasimenko V.I. *On the approaches to the derivation of the Boltzmann equation with hard sphere collisions*. Proc. Inst. Math. NAS of Ukraine, **10** (2) (2013) 71--95.
2. Gerasimenko V.I., Kornienko A.G. *The Boltzmann kinetic equation with correlations for hard sphere fluids*. Reports of NAS of Ukraine **3** (2015) 17--23.
3. Gerasimenko V.I., Gapyak I.V. *Hard sphere dynamics and the Enskog equation*. Kinet. Relat. Models, **5** (3) (2012) 459--484.
4. Borovchenkova M.S., Gerasimenko V.I. *On the non-Markovian Enskog equation for granular gases*. J. Phys. A: Math. Theor., **47** (3) (2014) 035001.

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ANALYTICAL AND NUMERICAL FLOCKING IN THE CUCKER-SMALE MODEL WITH NOISE AND DELAY

Abstract: We investigate a generalization of the Cucker-Smale model for collective animal behavior which, in contrast to the original model, incorporates two additional processes:

(i) stochasticity (imperfections) of individual behaviour; and

(ii) delayed responses of individuals to signals in their environment.

These effects are present in animal decision making, but are often neglected in modelling.

The new model is formulated as a system of delayed stochastic differential equations with multiplicative noise. We derive sufficient conditions for flocking by using a suitable Lyapunov functional. As a byproduct, we obtain a new result regarding the asymptotic behavior of delayed geometric Brownian motion. Finally, we present results of a series of systematic numerical simulations that not only illustrate the analytical results, but hint at a somehow surprising behavior of the system - namely, that an introduction of intermediate time delay may facilitate flocking.

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ON THE FOKKER PLANCK EQUATION IN MULTI-DIMENSIONAL BOUNDED DOMAINS

Abstract: We consider the kinetic Fokker-Planck equation in general multi-dimensional bounded domains with absorbing boundary condition. We will discuss the time global existence of a weak solution and its interior and boundary regularity. In particular, there has not been many results on the regularity of solutions when the spatial domain has a boundary. We will show that the solution is smooth away from the singular set and Holder continuous up to the singular set.

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NUMERICAL METHODS FOR MEAN FIELD EQUATIONS DERIVED FROM INTERACTING PARTICLE SYSTEMS

Abstract: The talk will present numerical methods for mean field hierarchies derived from interacting particle systems. This includes nonlinear kinetic Fokker-Planck equations and their hydrodynamic and diffusive approximations. We discuss Semi-Lagrangian and Arbitrary-Lagrangian-Eulerian methods based on a Finite-Volume and a mesh-free particle discretization. Asymptotic preserving methods for the transition between the equations of the hierarchy are also discussed. Applications to granular flow, swarming and pedestrian flow simulations are shown.

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GLOBAL MINIMIZATION WITH INTERACTING PARTICLE SYSTEMS

with R. Pinnau, C. Totzeck, O. Tse (TU Kaiserslautern) and Y.-P. Choi (Imperial College London)

Abstract: The task of minimizing a given function globally is challenging, especially in high-dimensional search space. One well-known idea is to employ agents that explore the search space while communicating amongst each other in some heuristic manner. Such "particle swarm optimization" (PSO) methods typically are discrete, second-order and include memory effects, which makes analysis very difficult.

In this talk, we develop a global minimization method that mimics central PSO ideas but at the same time bears resemblance to well-known kinetic aggregation equations. The particle system therefore possesses a formal mean-field limit to a partial differential equation, which gives the possibility of analytical treatment.

Ideally, one would like the method to show a concentration / consensus formation of particles at the global minimum, so blow-up effects are \emph{desirable} at the right location.

In order to illustrate the method and the possible insights from the mean-field equation, we present findings on the numerical performance and discuss first results and ongoing challenges in the analysis. Remarkably, the algorithm shows a competitive performance in standard test cases.

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NUMERICAL SIMULATION OF THE CROOKES RADIOMETER

Abstract: The Crookes radiometer is a small mill enclosed in a glass bulb containing a partial vacuum. The vanes of the mill rotate when exposed to light. This rotation is due to the thermal transpiration, an effect explained by the kinetic theory of gases, as shown by Reynolds and Maxwell.

In this talk, I will present a numerical method to simulate this Crookes radiometer. The main difficulty is to treat moving boundaries inside a rarefied gas as modeled by the Boltzmann equation. This is made with a numerical scheme on Cartesian grids with a cut-cell approach and adaptive mesh refinement. The method can easily handle complex geometries and can be used to make full 3D simulations.

This work is in collaboration with Guillaume Dechristé.

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DISCONTINUOUS GALERKIN METHODS FOR BOLTZMANN – POISSON MODELS OF ELECTRON TRANSPORT IN SEMICONDUCTORS

Jose A. Morales Escalante In collaboration with Irene M. Gamba, Armando Majorana, Yingda Cheng, Chi-Wang Shu and James R. Chelikowsky.

Abstract: We shall discuss the implementation of Discontinuous Galerkin (DG) methods on Boltzmann - Poisson (BP) systems of electron transport in semiconductor devices at nano scales.

First we consider the implementation of full energy bands given by Empirical Pseudopotential Methods (EPM) in DG-BP schemes by means of a spherical average EPM band, as a mid-point between a radial band model and a full anisotropic band structure. We compare the computational prediction of physical observables, via moments, with the ones related to traditionally used analytical conduction bands such as Parabolic and Kane band models.

We also consider the mathematical and numerical modeling of Reflective Boundary Conditions in 2D devices and their implementation in DG-BP schemes. We study the specular, diffusive and mixed reflection BC on physical boundaries of the device for the modeling of surface roughness, comparing the influence of these different reflection cases in the computational prediction of moments close to the boundaries and their associated scale.

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MODEL PREDICTIVE BOLTZMANN CONTROL

Abstract: In this talk we survey some recent results on the control of complex socio-economic systems composed by a large number of agents. We focus in particular on constrained alignment models and investigate model predictive control techniques in the mean-field and Boltzmann limits. Connections with continuous control based on Riccati equations are also presented. Finally the presence of random inputs in the system is considered and the need to control instabilities is discussed. Several numerical results illustrate the different approaches.

References:

[1] G.Albi, M.Herty, and L.Pareschi. Kinetic description of optimal control problems in consensus modeling. Comm. Math. Sci., 13, 1407-1429, 2015

[2] G.Albi, L.Pareschi, and M.Zanella. Boltzmann type control of opinion consensus through leaders. Phil. Trans. A Math. Phys. Eng. Sci., 13:372(2028), 2014

[3] M.Herty, L.Pareschi, S.Steffensen. Mean-field control and Riccati equations. Network and Heterogeneous Media, 2015 to appear

[4] G.Albi, L.Pareschi, and M.Zanella. Uncertainty quantification in control problems for flocking models, Mathematical problems in Engineering, 2015 to appear.

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THE VLASOV-POISSON SYSTEM FOR STELLAR DYNAMICS IN SPACES OF CONSTANT CURVATURE

Abstract: We obtain a natural extension of the Vlasov-Poisson system for stellar dynamics to spaces of constant Gaussian curvature: the unit sphere and the unit hyperbolic sphere. These equations can be easily generalized to higher dimensions, but for simplicity we focus on the 1-dimensional problem, i.e. when the particles move on a geodesic. In this case, we prove the global existence of solutions and show that the solutions of the linearized system are stable in the sense of Landau damping.

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PRICE FORMATION: FROM BOLTZMANN TO MEAN FIELD MODELS

Abstract: In 2007 Lasry & Lions introduced a simple mean field model for the dynamical formation of a price. The model consists of a system of parabolic equations for the trader densities (as functions of the bid-ask price), with the agreed price entering as a free boundary. The authors motivated the model using mean field game theory, but the detailed microscopic origin remained unclear.

In this talk we provide a simple agent based trade model with standard stochastic price fluctuations together with discrete trading events. By modeling trading events between vendors and buyers as kinetic collisions we obtain a Boltzmann-type model for the densities. We prove rigorously that in the limit of large trading frequencies, the proposed Boltzmann model converges to the Lasry and Lions mean field model. We also analyze other asymptotics beyond the scales that the model can describe and illustrate our analytical results with numerical simulations.

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