CMO-BIRS Workshop: Modern Techniques in Discrete Optimization: Mathematics, Algorithms and Applications
November 1-6, 2015

MEALS

*Breakfast: 7:30 – 9:00 am, Restaurant Hotel Hacienda Los Laureles, Monday–Friday
*Lunch: 13:30 – 15:00 pm, Restaurant Hotel Hacienda Los Laureles, Monday–Friday
*Dinner: 19:00 – 21:00 pm, Restaurant Hotel Hacienda Los Laureles, Monday–Thursday
*Dinner: 19:30 – 22:00 pm, Restaurant Hotel Hacienda Los Laureles, Sunday only
*Continuous Coffee Breaks: Conference Room San Felipe, Hotel Hacienda Los Laureles

MEETING ROOMS

All lectures will be held in the Conference Room San Felipe at Hotel Hacienda Los Laureles. An LCD projector, laptop, document camera and blackboards are available for presentations.

SCHEDULE

Sunday	
14:00	Check-in begins (front desk at your assigned hotel - open 24 hours)
19:30–22:00	Dinner, Restaurant Hotel Hacienda Los Laureles
20:30	Informal gathering Hotel Hacienda Los Laureles; a welcome drink will be served by the hotel.
Monday	
7:30–8:45	Breakfast
8:45–9:00	Introduction and Welcome
9:00–10:00	Long lecture: Daniel Bienstock (Columbia Univ), *Exploiting structured sparsity in mixed-integer polynomial optimization*
10:00–10:30	Coffee break
10:30–11:00	Short lecture: Antoine Deza (McMaster Univ.) *On the diameter of lattice polytopes*
11:00–11:15	Discussion break
11:15–11:45	Short lecture: Gabriela Araujo (UNAM), *The cage problem*
11:45–12:00	Discussion break
12:00–12:30	Short lecture: Walter Morris (George Mason Univ.) *A directed Steinitz theorem for oriented matroid programming*
12:30–12:45	Discussion break
12:45–13:30	Long lecture: Shabbir Ahmed (Georgia Tech), *Exact Augmented Lagrangian Duality in Mixed Integer Linear Programming*
13:30–13:35	Group photo
13:35–15:00	Lunch
15:00–17:30	Afternoon for self-organized discussions
18:00–19:00	Problem Session (all encouraged to present)
19:00-21:00	Dinner

Tuesday

7:30–9:00	Breakfast
9:00–10:00	Long lecture: Thomas Rothvoss (Univ. Washington) , *Constructive discrepancy minimization for convex sets*
10:00–10:30	Coffee break
10:30–11:00	Short lecture: Criel Merino (UNAM) , *On zeros of the characteristic polynomial of representable matroids of bounded tree-width.*
11:00–11:15	Discussion break
11:15–11:45	Short lecture: Tamon Stephen (Simon Fraser Univ), *Polyhedral aspects of circuit-based pivoting algorithms*
11:45–12:00	Discussion break
12:00–12:30	Short lecture: Alejandro Torrielo (Georgia Tech), *Relaxations for a Dynamic Knapsack Problem*
12:30–12:45	Discussion break
12:45–13:30	Long lecture: Dorit Hochbaum (Univ. California, Berkeley), *Effective combinatorial algorithms for image segmentation and data mining*
13:30–15:00	Lunch
15:00–16:00	Long lecture: Amitabh Basu (John Hopkins Univ), *An introduction to cut generating functions*
16:00–19:00	Afternoon for self-organized discussions
19:00-21:00	Dinner

Wednesday

7:30–9:00	Breakfast
9:00–1:00	Self-organized visit to Monte Alban
1:00–15:30	Lunch on your own Downtown
16:30–17:00	Short lecture: Roger Z. Rios-Mercado (UANL), *Districting Problems: Models, Algorithms and Research Trends*
17:00–17:15	Discussion break
17:15–17:45	Short lecture: Francisco Zaragoza (UAM, Atzcapozalco), *Traveling Repairman Problem on a Line with Unit Time Windows*
17:45–18:00	Discussion break
18:00–18:30	Short lecture: Greg Blekherman (Georgia Tech), *Spectrahedral Cones with rank 1 extreme rays*
18:30–18:45	Discussion break
18:45–19:15	Short lecture: Juan Pablo Vielma (MIT), *Embedding Formulations, Complexity and Representability for Unions of Convex Sets*
19:00-21:00	Dinner

Thursday

7:30–9:00	Breakfast
9:00–10:00	Long Lecture: Sebastian Pokutta (Georgia Tech) *Extended formulations: the impressive power of LPs and SDPs*
10:00–10:30	Coffee break
10:30–11:00	Short lecture: Justo Puerto (Univ of Seville) *New results on k-sum and ordered median combinatorial optimization problems*
11:00–11:15	Discussion break
11:15–11:45	Short lecture: Oktay Gunluk (IBM) *Cutting planes from extended LP formulations*
11:45–12:00	Discussion break
12:00–12:30	Short lecture: Kurt Anstreicher (Univ. of Iowa), *Kronecker Product Constraints for Semidefinite Optimization*
12:30–12:45	Discussion break
12:45–13:30	Long lecture: Santanu S. Dey (Georgia Tech) *Analysis of sparse cutting-plane for sparse MILPs with applications to stochastic MILPs*
13:30–15:00	Lunch
15:00–16:00	Problem session (all encouraged to present)
16:00–19:00	Free Discussion
19:00-21:00	Dinner

Friday

7:30–9:00	Breakfast
9:00–10:00	Long lecture: Levent Tunçel (Univ of Waterloo), *Elementary Polytopes, their Lift-and-Project Ranks and Integrality Gaps*
10:00–10:15	Discussion Break
10:15–10:45	Short lecture: Matthias Köppe (Univ of California, Davis), *Gomory-Johnson's infinite group relaxation: Algorithmic aspects*
10:45–11:00	Closing
11:00–...	Survivors self organize
Checkout (by 12 noon)	

** 5-day workshop participants are welcome to use Hotel Hacienda Los Laureles facilities until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

ABSTRACTS
(in alphabetic order by speaker surname)

Speaker: **Shabbir Ahmed** Georgia Tech, GA, USA
Title: *Exact Augmented Lagrangian Duality in Mixed Integer Linear Programming*
Abstract: We consider the general mixed integer (linear) programming (MIP) problem

$$z^{\mathrm{IP}} := \inf\{\boldsymbol{c}^\top \boldsymbol{x} | \boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}, \boldsymbol{x} \in X\}, \tag{1}$$

and its augmented Lagrangian dual (ALD)

$$z_\rho^{\mathrm{LD+}} := \sup_{\boldsymbol{\lambda} \in \Re^n} \inf_{\boldsymbol{x} \in X} \{\boldsymbol{c}^\top \boldsymbol{x} + \boldsymbol{\lambda}^\top (\boldsymbol{b} - \boldsymbol{A}\boldsymbol{x}) + \rho\psi(\boldsymbol{b} - \boldsymbol{A}\boldsymbol{x})\} \tag{2}$$

where X is a mixed integer linear set, ρ is a given positive scalar, and $\psi(\cdot)$ is an augmenting function with $\psi(\mathbf{0}) = 0$ and $\psi(\boldsymbol{u}) > 0$ for all $\boldsymbol{u} \neq \mathbf{0}$. ALD provides a lower bound for the problem (1), i.e. $z_\rho^{\mathrm{LD+}} \leq z^{\mathrm{IP}}$, for all $\rho > 0$.

We consider non-negative level bounded augmenting functions in ALD for solving MIPs. Because of the non-convexity in MIP (1), a non-zero duality gap may exist, that is $z^{\mathrm{IP}} - z_\rho^{\mathrm{LD+}} > 0$. Recently, Boland and Eberhard showed that in ALD for MIPs, with a specific class of nonnegative convex augmenting functions, $\lim_{\rho \to \infty} z_\rho^{\mathrm{LD+}} = z^{\mathrm{IP}}$ holds. They also proved that if X is a finite set (e.g. a bounded pure IP), then there exists a finite penalty coefficient which closes the duality gap. In this work, we significantly generalize the results by Boland and Eberhard. Our main contributions are as follows:

1. We first provide a primal characterization for the ALD of an MIP. This is an alternative characterization to the one provided by Boland and Eberhard. Using this characterization, the ALD of an MIP can be viewed as a traditional Lagrangian dual (LD) in a lifted space.

2. We give an alternative proof for the asymptotic zero duality gap property of ALD for MIPs when the penalty coefficient is allowed to go to infinity. This was first proved by Boland and Eberhard.

3. We prove that ALD using any norm as the augmenting function with a sufficiently large but *finite* penalty coefficient closes the duality gap for general MIPs. This generalizes the result by Boland and Eberhard from the case of pure integer programming with a bounded feasible region to general MIPs with unbounded feasible regions.

4. Using our primal characterization, we also present an example where ALD with a quadratic augmenting function is not able to close the duality gap for any finite penalty coefficient. This is joint work with Mohammad Javad Feizollahi and Andy Sun.

Speaker: **Kurt Anstreicher** (University of Iowa)
Title: *A Kronecker Product Constraints for Semidefinite Optimization*
Abstract: We consider a new class of constraints for semidefinite optimization based on Kronecker products of semidefinite matrices. These constraints generalize the well-known RLT and SOC-RLT constraints. We show that the Kronecker product constraint generated by two second-order cone (SOC) constraints implies all of the SOC-RLT constraints that can be generated from the two SOC constraints. The Kronecker product constraint grows rapidly in size, but in the case of two SOC constraints has a block structure that permits efficient generation of valid linear inequalities (cuts). Computational results on difficult instances of

the two-trust-region subproblem (TTRS) shows that the approach utilizing cuts from a Kronecker product constraint improves on the best previous results for computable convex relaxations of TTRS.

Speaker: **Gabriela Araujo** (UNAM)
Title: *The Cage Problem*
Abstract: A graph is k-regular if all the vertices have degree equal to k. The *girth* of a graph, denoted by g, is the number of edges in the smallest cycle(s). The Cage Problem is related to the existence and construction of regular graphs with a fixed girth and the minimum number of vertices possible.

A $(k; g)$-graph is a k-regular graph of girth g, and a $(k; g)$-cage is a $(k; g)$-graph of minimum order. Cages were introduced by Tutte in 1947 and their existence was proved by Erdös and Sachs in 1963 for all integer values of k and g with $k \geq 2$. Since then most work carried out has focused on constructing the ones with the smallest number of vertices.

The author has worked in this problem since 2006. She has constructed, jointly with several coauthors, different families of "small" $(k; g)$-graphs for many values of k and g. In this work, she will exhibit the main techniques for constructing these graphs; using finite geometries. She will also exhibit the minimal values for the upper bounds for the order of the cages known up to date.

Specially, related with the topic of this Conference, the author will talk about a specific problem, the construction of $(k; 5)$-minimal cages using projective planes (finite geometries), and she will explain how it relates to computer applications.

Speaker: **Amitabh Basu** The Johns Hopkins University, MD, USA
Title: *An introduction to Cut-generating functions*
Abstract: Cut-generating functions are a means to have "a priori" formulas for generating cutting planes for general mixed-integer optimization problems. Let S be a closed subset of $\mathbb{R}^n$ with $0 \notin S$. Consider the following set, parametrized by matrices R, P:

$$X_S(R, P) := \left\{(s, y) \in \mathbb{R}^k_+ \times \mathbb{Z}^\ell_+ \, : \, Rs + Py \in S\right\}, \tag{3}$$

where $k, \ell \in \mathbb{Z}_+, n \in \mathbb{N}$, $R \in \mathbb{R}^{n\times k}$ and $P \in \mathbb{R}^{n\times \ell}$ are matrices. Denote the columns of matrices R and P by $r_1, \ldots, r_k$ and $p_1, \ldots, p_\ell$, respectively. We allow the possibility that $k = 0$ or $\ell = 0$ (but not both). This general model contains as special cases classical optimization models such as *mixed-integer linear optimization* and *mixed-integer convex optimization.*

Given $n \in \mathbb{N}$ and a closed subset $S \subseteq \mathbb{R}^n$ such that $0 \notin S$, a *cut-generating pair* (ψ, π) for S is a pair of functions $\psi, \pi : \mathbb{R}^n \to \mathbb{R}$ such that

$$\sum_{i=1}^{k} \psi(r_i)s_i + \sum_{j=1}^{\ell} \pi(p_j)y_j \geq 1 \tag{4}$$

is a valid inequality (also called a *cut*) for the set $X_S(R, P)$ for every choice of $k, \ell \in \mathbb{Z}_+$ and for all matrices $R \in \mathbb{R}^{n\times k}$ and $P \in \mathbb{R}^{n\times \ell}$. Cut-generating pairs thus provide cuts that separate 0 from the set $X_S(R, P)$. We emphasize that cut-generating pairs depend on n and S and do *not* depend on k, ℓ, R and P. There is a natural partial order on the set of cut generating pairs; namely, $(\psi', \pi') \leq (\psi, \pi)$ if and only if $\psi' \leq \psi$ and $\pi' \leq \pi$. The minimal elements under this partial ordering are called *minimal cut-generating pairs.*

Efficient procedures for cut-generating pairs. Several deep structural results were obtained by Johnson about minimal cut-generating functions for S when S is a translated lattice, i.e., $S = b + \mathbb{Z}^n$ for some $b \in \mathbb{R}^n \setminus \mathbb{Z}^n$. However, a major drawback is that the theory developed is abstract and difficult to use from a computational perspective. A recent approach has been to restrict attention to a specific class of minimal cut-generating pairs for which we can give computational procedures to compute the values $\psi(r_i)$ and $\pi(p_j)$. We show how this is done when S is a translated lattice intersected with a polyhedron, i.e., $S = (b + \mathbb{Z}^n) \cap Q$ for some vector $b \in \mathbb{R}^n \setminus \mathbb{Z}^n$ and some rational polyhedron Q.

Given such a set $S \subseteq \mathbb{R}^n$, define $W_S := \mathbb{Z}^n \cap \text{lin}(\text{conv}(S))$. A convex set B is called S-free if $\text{int}(B) \cap S = \emptyset$. A *maximal* S-free convex set is an S-free convex set that is inclusion wise maximal. A well-known theorem others). It is known a maximal S-free convex set B containing the origin in its interior is a polyhedron given by

$$B = \{r \in \mathbb{R}^n : a_i \cdot r \leq 1 \;\; i \in I\}. \tag{5}$$

THEOREM 1. *Define the following pair of functions associated with B:*

$$\psi_B(r) = \max_{i \in I} a_i \cdot r, \quad \pi_B(r) = \inf_{w \in W_S} \psi(r + w) \tag{6}$$

(ψ_B, π_B) is a valid cut-generating pair. Moreover, the pair is "partially" minimal: for every cut-generating pair $(\psi, \pi) \leq (\psi_B, \pi_B)$, we must have $\psi = \psi_B$.

Thus, for every maximal S-free convex set B, (6) gives formulas to compute with the corresponding cut-generating pair (ψ_B, π_B). However, because of the partial minimality of (ψ_B, pi_B), it may be the case that there exists a pair (ψ, π) with $\pi \leq \pi_B$ and $\pi(r) < \pi_B(r)$ for some $r \in \mathbb{R}^n$. **The main question of this talk is:**

QUESTION: Let $S = (b + \mathbb{Z}^n) \cap Q$ with $b \in \mathbb{R}^n \setminus \mathbb{Z}^n$ and a rational polyhedron Q. Given a maximal S-free convex set B (5), decide if (ψ_B, π_B) is minimal.

Speaker: **Daniel Bienstock** , (Columbia University)
Title: *Exploiting structured sparsity in mixed-integer polynomial optimization*
Abstract: Joint work with Gonzalo Munoz. Many ideas in (continuous) polynomial optimization algorithms make use of the structural sparsity of the intersection graph of the constraints (Waki et al, Lasserre et al). Often this leads to e.g. sum-of-squares or semidefinite relaxations whose solution is made more efficient by leveraging the sparsity; however concrete convergence results are scarce. In this talk we describe linear programming approximations to mixed-integer polynomial optimization problems where the intersection graph of the constraints has fixed tree-width. The LP formulations, given epsilon ¿ 0, are polynomially large in the problem data and in ϵ^{-1}, and provably attain epsilon-optimality and feasibility guarantee. As a consequence we obtain an LP-based polynomial-time approximation algorithm for the ACOPF problem on graphs with bounded tree-width.

Speaker: **Greg Blekherman** (Georgia Tech, USA)
Title: *Spectrahedral Cones with rank 1 extreme rays*
Abstract: A spectrahedral cone C is a slice of the cone of positive semidefinite matrices with a linear subspace L. The ranks of extreme rays of spectrahedral cones have been a subject of extensive study. It is natural to ask for what subspaces L do all of the extreme rays of C have rank 1? When L is a union of coordinate subspaces the answer was given by Agler-Helton-McCullough-Rodman. It turns out that this question has an unexpected connection to algebraic geometry and we will present a full classification of such spectrahedral cones based on the classification of small reduced schemes by Eisenbud-Green-Hulek-Popescu. No knowledge of scheme theory will be necessary for the talk.

Speaker: **Antoine Deza** (McMaster University, Canada)
Title: *On the diameters of Lattice polytopes*
Abstract: Finding a good bound on the maximal diameter D(d,n) of the vertex-edge graph of a polytope in terms of its dimension d and the number of its facets n is one of the basic open questions in polytope theory. The Hirsch conjecture, formulated in 1957 states that D(d,n) is at most n-d. While the conjecture was disproved by Santos in 2010, it is known to hold in small dimensions along with other specific pairs of d and n. However, the asymptotic behavior of D(d,n) is not well understood: the best upper bound is

quasi-polynomial. The behavior of D(d,n) is not only a natural question of extremal discrete geometry, but is historically closely connected with the theory of the simplex method. We present older and recent results dealing with the diameter of lattice polytopes.

Speaker: **Santanu Dey** (Georgia Tech, USA)
Title: *Analysis of sparse cutting-plane for sparse MILPs with applications to stochastic MILPs*
Abstract: While numerous families of cutting-planes have been studied for mixed integer linear programs (MILPs), significantly lesser understanding has been obtained on the very important question of cutting-plane selection from a theoretical perspective. State-of-the-art MILP solvers bias the selection of cutting-planes towards sparse cuts: This is a natural choice since solving a MILP involves solving many linear programs (LP) and LP solvers can take advantage of sparse constraint matrices. In a recent work with Molinaro and Wang we presented a geometric analysis of the quality of sparse cutting-planes as a function of the number of vertices of the integer hull, the dimension of the polytope and the level of sparsity. We pursue this question of understanding the strength of sparse cutting-planes using completely different techniques, so that we are also able to incorporate the information that most real-life MILP formulations have sparse constraint matrices. This talk presents our recent work on the topic.

Speaker: **Oktay Gunluk** (IBM Research)
Title: *Cutting planes from extended LP formulations*
Joint with Bodur and Dash. Given a mixed-integer set defined by linear inequalities and integrality requirements on some of the variables, we consider extended formulations of its continuous (LP) relaxation and study the effect of adding cutting planes in the extended space. In terms of optimization, extended LP formulations do not lead to better bounds as their projection onto the original space is precisely the original LP relaxation. However, adding cutting planes in the extended space can lead to stronger bounds. In this paper we show that for every 0-1 mixed-integer set with n integer and k continuous variables, there is an extended LP formulation with 2n+k-1 variables whose elementary split closure is integral. The proof is constructive but it requires an inner description of the LP relaxation. We then extend this idea to general mixed-integer sets and construct the best extended LP formulation for such sets with respect to lattice-free cuts. We also present computational results on the two-row continuous group relaxation showing the strength of cutting planes derived from extended LP formulations.

Speaker: **Dorit Hochbaum** (Univ. of California, Berkeley, USA)
Title: *Effective combinatorial algorithms for image segmentation and data mining*
Abstract: We present a model for clustering which combines two criteria: Given a collection of objects with pairwise similarity measure, the problem is to find a cluster that is as dissimilar as possible from the complement, while having as much similarity as possible within the cluster. The two objectives are combined either as a ratio or with linear weights. The ratio problem, and its linear weighted version, are solved by a combinatorial algorithm within the complexity of a single minimum s,t-cut algorithm. We call this problem "the normalized cut prime" (NC') as it is closed related to the NP-hard problem of normalized cut.

The relationship of NC' to normalized cut is generalized to a problem we call "q-normalized cut". It is shown that the spectral method that solves for the Fielder eigenvector of a related matrix is a continuous relaxation of the problem. In contrast, the generalization of the combinatorial algorithm solves a discrete problem resulting from a relaxation of a single sum constraint. We study the relationship between these two relaxations and demonstrate a number of advantages for the combinatorial algorithm. These advantages include a better approximation, in practice, of the normalized cut objective for image segmentation benchmark problems. Time permitting, I will discuss the application of NC', as a supervised machine learning technique, to data mining, and its comparison to leading machine learning techniques on datasets selected from data mining benchmark.

Speaker **Matthias Köppe** (Univ. of California, Davis, USA)
Title: *Gomory-Johnson's infinite group relaxation: Algorithmic aspects*

The infinite group problem was introduced 42 years ago by Ralph Gomory and Ellis Johnson in their groundbreaking papers titled "Some continuous functions related to corner polyhedra I, II". The technique, investigating strong relaxations of integer linear programs by convexity in a function space, has at times been dismissed as "esoteric". Now we recognize the infinite group problem as a technique which was decades ahead of its time, providing the first "cut-generating function" approach to integer programming. It may be the key to today's pressing need for stronger, "multi-row" cutting plane approaches.

I survey the recent progress on the problem, focusing on algorithmic aspects, such as the automatic extremality test for cut generating functions in the Gomory-Johnson model, its implementation in software, and ongoing work on automatic discovery and proof of cutting plane theorems in the Gomory-Johnson model. (based on joint work with A. Basu, R. Hildebrand, R. La Haye, M. Molinaro, Q. Louveaux, Y. Zhou)

Speaker **Criel Merino** (UNAM, Mexico)
Title: *On zeros of the characteristic polynomial of representable matroids of bounded tree-width.*
Abstract: Traditionally, the focus from a graph theory perspective about the chromatic polynomial has been to find the positive integer roots λ, which correspond to the graph not being properly colourable with λ colours. A growing body of work has begun to emerge in recent years more concerned with the behaviour of real or complex roots of the chromatic polynomial. Perhaps one of the outstanding open questions concerning real zeros is to determine tight bounds on the largest real zero of the chromatic polynomial. One such bound was given by A. Sokal, and more recently by F. M. Dong, and depends on the maximum vertex degree of the graph. The corresponding invariant in Matroids is the characteristic polynomial. We prove that, for any prime power q and constant k, the characteristic polynomial of any loopless, $GF(q)$-representable matroid with tree-width k has no real zero greater than $q^{k?1}$.

Speaker: **Walter Morris** (George Mason Univ. USA)
Title: *A directed Steinitz theorem for oriented matroid programming.*

Abstract: Holt and Klee proved that if P is a d-dimensional polytope and f is a linear function on P that is not constant on any edge of P, there are d independent monotone paths from the source to the sink of the digraph defined by the vertices and edges of P directed according to the directions of increase of f. Mihalisin and Klee proved that every orientation of the graph of a 3-polytope that is acyclic and admits 3 independent monotone paths from the source to the sink is obtained from some 3-polytope P and some linear function f on P. We prove analogs of Mihalisin and Klee's theorem and the 3 and 4-dimensional versions of Holt and Klee's theorem for oriented matroid programs. Here acyclicity is replaced by the requirement that there be no directed cycle contained in a face of the polytope.

Speaker: **Sebastian Pokutta** (Georgia Tech, USA)
Title: *Extended Formulations: the expressive power of LPs and SDPs*
Linear and semidefinite programming are two core optimization paradigms with many important applications in mathematics, engineering, and business. However, the expressive power of these modeling paradigms is only partially understood so far and *extended formulations* are a powerful and natural tool to analyze the possibilities and limitations of linear and semidefinite programming formulations. More precisely, extended formulations are concerned with studying the optimal representation of a combinatorial optimization problem in terms of LPs, SDPs, or alternative conic programming paradigms. An extended formulation is a higher dimensional description of the problem utilizing auxiliary variables. For the sake of brevity of this abstract we will confine ourselves to linear programs, however the theory applies more broadly to many other paradigms, such as e.g., semidefinite programming, which will be also at least partially covered in the talk.

The main goal that we will be concerned with is to reduce the number of required inequalities in a linear programming formulation by representing a given optimization problem in slightly higher dimensional space or ruling out the existence of such formulations. Recently, extended formulations gained significant interest due to fundamental questions in optimization and complexity theory that are closely related to the notion of extended formulations. In fact, extended formulations provide an alternative measure of 'complexity', which is independent of P vs. NP: we count the *number* of required inequalities and the encoding of the coefficients is disregarded. This distinctive criterion makes extended formulations very attractive as the obtained statements are not subject to any complexity theoretic assumptions and it has been argued that the resulting notion of complexity is more in line with how we solve linear programs. Moreover, this notion of complexity might also provide supporting evidence for several conjectures in complexity theory.

More formally, our setup will be the following. Let $P = \{x \mid Ax \leq b\} \subseteq \mathbb{R}^n$ be a polytope representing a combinatorial optimization problem of interest. A polytope $Q = \{x \mid Ex \leq d\} \subseteq \mathbb{R}^m$ with $m \geq n$ is called an *extension of* Q if there exists a linear map π with $P = \pi(Q)$. The smallest number of inequalities required in any extension of P is called the *extension complexity* $\mathrm{xc}(P)$ *of* P. Any extension Q can be used as a surrogate to optimize over P and thus we are interested in finding the smallest possible extension. We therefore ask:

What is the smallest number of inequalities required
in any extension Q of P?

Put differently, we aim for determining the extension complexity of P. In many cases using an extended formulation can lead to an exponential saving in terms of the number of inequalities, i.e., a polytope P with an *exponential number* of inequalities in the description $Ax \leq b$ can be expressed in slightly higher dimensional space with a *polynomial* number of inequalities, allowing for efficient optimization over P via linear programming (provided the coefficients are small). Examples include the Spanning Tree Polytope as well as the extended formulations for the regular polygon, which can be used to approximate the second-order cone efficiently. In several other important (and surprising cases), such as e.g., the Traveling Salesman Polytope and Matching Polytope it can be shown that such compact formulations cannot exist.

The theory also naturally extends to approximate formulations and many surprising examples have been recently obtained. For example, it was shown that the MaxCut Problem cannot be approximated better than 1/2 with a polynomial size linear program. Also, the VertexCover Problem cannot be approximated better than a factor of 2 using a polynomial size linear program.

In this talk I will provide an introduction to extended formulations and survey many of the aforementioned results in extended formulations, both in the linear and the semidefinite setting. I will also lay out a reduction framework for establishing upper and lower bounds for the size of exact and approximate LP and SDP formulations. This framework allows for surprisingly simple and convenient analyzes without relying on any heavy machinery, making extended formulations very accessible without requiring any in-depth prior knowledge of those results establishing the base hardness. I will conclude with various open problems both in the exact and approximate as well as linear and semidefinite case.

Speaker:**Justo Puerto (Univ. de Sevilla, Spain)**
Title:*New results on k-sum and ordered median combinatorial optimization problems*
Abstract: In this talk, we address the continuous, integer and combinatorial k-sum and ordered median optimization problems. We analyze different reformulations of these problems that allow to solve them through the minimization of minisum optimization problems. This approach provides a general tool for solving ordered median optimization problems and improves the complexity bounds of many ad-hoc algorithms previously developed in the literature for particular versions of these problems.

Speaker : **Roger Z. Rios-Mercado** Univ. Autónoma de Nuevo León
Title*Districting Problems: Models, Algorithms and Research Trends*
Abstract: Districting or territory design involves locational decisions where a given set of basic or geographic units must be partitioned so as to optimize some performance measure subject to pre-specified planning requirements. Typical criteria usually sought are territory compactness, connectivity, balancing, similarity with existing plan, etc. Depending on the particular application, different models or dispersion measure

can be used. In this talk we will give an overview of the main elements involving districting decisions, and present some of the models, and solution algorithms (exact and heuristic) that have been developed for particular districting applications. We will close the talk discussing future research directions in this area.

Speaker: **Thomas Rothvoss** Univ. of Washington, Seattle, USA
Title: *Constructive discrepancy minimization for convex sets*

Abstract: A classical theorem of Spencer shows that any set system with n sets and n elements admits a coloring of discrepancy $O(n^1/2)$. Recent exciting work of Bansal, Lovett and Meka shows that such colorings can be found in polynomial time. In fact, the Lovett-Meka algorithm finds a half integral point in any "large enough" polytope. However, their algorithm crucially relies on the facet structure and does not apply to general convex sets. We show that for any symmetric convex set K with measure at least exp(-n/500), the following algorithm finds a point y in $K \cap [-1,1]^n$ with Omega(n) coordinates in $\{-1,+1\}$: (1) take a random Gaussian vector x; (2) compute the point y in $K \cap [-1,1]^n$ that is closest to x. (3) return y. This provides another truly constructive proof of Spencer's theorem and the first constructive proof of a Theorem of Gluskin and Giannopoulos.

Speaker: **Tamon Stephen** Simon Fraser Univ. Canada
Title: *Polyhedral aspects of circuit-based pivoting algorithms*

Abstract: We consider prospects for augmented pivoting algorithms for linearly constrained optimization. Here, in addition to the local edge directions used in the simplex method, a discrete set of additional directions are available. Some of these pass into the interior of the feasible region or of mid-dimensional faces.

We focus on the case where the available directions are the *circuits* or *elementary vectors*, i.e. the support minimal solutions to the homogenization of the equations defining the feasible region. These can also be thought of as potential edge directions of the system under varying right-hand sides of the equations. Once a direction is chosen, it is followed as far as feasibility allows.

A necessary condition for the existence of high-quality pivoting algorithms of this type is the existence of short paths between the vertices of the feasible region in the sense of using only a small number of these pivots and augmentations. This leads to the notion of the *circuit diameter* of a polyhedron P, which is number of pivots in the longest minimal path between any pair of vertices in P. The circuit diameter is an analogue of the *combinatorial diameter* of a polytope, which is the longest minimal path using only edge moves. Thus the circuit diameter is a lower bound for the combinatorial diameter, and the bound is tight for key families.

Borgwardt, Finhold and Hemmecke (2014) investigated circuit diameter and showed that dual transportation polyhedra have a very low circuit diameter, lower than their combinatorial diameter. They asked if the Hirsch bound of $f - d$ (the number of facets of the polytope minus its dimension) which was conjectured as an upper bound on the combinatorial diameter, might hold for the combinatorial diameter.

We show that some known non-Hirsch polyhedra, notably the Klee-Walkup polyhedron, are not counterexamples to this circuit Hirsch conjecture. We survey current work and open questions.

Speaker: **Alejandro Torrielo** (Georgia Tech, USA)
Title: *Relaxations for a Dynamic Knapsack Problem*

Joint work with Daniel Blado and Weihong Hu. We consider a dynamic version of the classical knapsack problem with the following formulation. Let $N := \{1, \ldots, n\}$ be a set of items. For each item $i \in N$ we have a non-negative, independent random variable A_i with known distribution representing its size, and a deterministic value $c_i > 0$. We have a knapsack of deterministic capacity $b > 0$, and we would like to maximize the expected total value of inserted items. An item's size is realized when we choose to insert it, and we receive its value only if the knapsack's remaining capacity is greater than or equal to the realized size. Given any remaining capacity $s \in [0, b]$, we may choose to insert any available item, and the decision is irrevocable. If the insertion is unsuccessful, i.e. the realized size is greater than the remaining capacity, the process terminates.

This model and other like it have applications in scheduling, equipment replacement and machine learning, to name a few examples. They also generally reflect some trends in optimization research and its various application, which have focused attention on models in which uncertain data is not revealed at once after an initial decision stage, but rather is dynamically revealed over time based on the decision maker's choices.

The deterministic knapsack is a special case, so this problem is NP-hard, and some variants are known to be PSPACE-hard. Because the decision maker can choose any item to insert based on remaining capacity, a solution is not simply a subset of items, but rather a policy that prescribes what item to insert under all possible circumstances. Research on the model has therefore studied heuristic policies and tractable relaxations. Our focus is mostly on the latter, deriving mathematical programming relaxations that can be solved efficiently, and which can be used to design high-quality heuristics. Specifically:

1. We introduce a semi-infinite relaxation for the problem under arbitrary item size distributions, based on an affine value function approximation of the linear programming encoding of the problem's dynamic program. We show that the number of constraints in this relaxation is at worst countably infinite, and is polynomial in the input for distributions with finite support.

2. When item sizes have integer support, we show that non-parametric value function approximation gives the strongest known relaxation from the literature, which has pseudo-polynomially many variables and constraints.

3. We theoretically and empirically compare these relaxations to others from the literature and show that both are quite tight. In particular, our new relaxation is notably tighter than a variety of benchmarks and compares favorably to the theoretically stronger pseudo-polynomial relaxation when this latter bound can be computed.

Time permitting, we also discuss future work and open questions motivated by our results, including the theoretical worst-case gap of our new relaxation, the possible strengthening of our relaxations, their asymptotic behavior as the number of items grows, and others.

Speaker: **Levent Tunçel** (University of Waterloo, Canada)
Title:*Elementary Polytopes, their Lift-and-Project Ranks and Integrality Gaps*

Abstract: We consider some elementary polytopes and study the performance of some of the strongest lift-and-project operators in computing the convex hull of integral points inside the given elementary polytope. Our study includes the analysis of the number of major iterations required as well as an analysis of the changes in the integrality gaps throughout these major iterations. This talk is based on joint work with Y. H. (Gary) Au.

Speaker: **Juan Pablo Vielma** (MIT, USA)
Title: *Embedding Formulations, Complexity and Representability for Unions of Convex Sets*

We consider strong Mixed Integer Programming (MIP) formulations for a disjunctive constraint of the form

$$x \in \bigcup\nolimits_{i=1}^{n} C_i \tag{7}$$

where $\{C_i\}_{i=1}^n \subseteq \mathbb{R}^d$ is a finite family of compact convex sets. MIP formulations for (7) can can be divided into two classes depending on their strength and types of auxiliary variables. The first class corresponds to extended formulations that use both 0-1 and continuous auxiliary variables. Standard versions of such extended formulations have sizes that are linear on appropriate size descriptions of the convex sets (e.g. number of linear, quadratic or conic constraints) and have continuous relaxations with extreme points that naturally satisfy the integrality constraints on the 0-1 variables (such formulations are usually denoted *ideal* and are as strong as possible). Extended formulations for polyhedral sets have been introduced by Balas, Jeroslow and Lowe, for conic representable sets by Ben-Tal, Helton, Nemirovski and Nie and for sets described through non-linear inequalities by Ceria, Merhotra, Soares and Stubs. The second class corresponds to non-extended formulations that only use the 0-1 variables that are strictly necessary for a valid formulation. Standard versions of such non-extended formulations are also linear sized, but are often significantly weaker than their extended counterparts. Non-extended formulations include big-M type constraints and ad-hoc formulations for specially structured polyhedral sets

A common feature of both classes is the use of n 0-1 variables that are constrained to add up to one. However, in the polyhedral setting different uses of 0-1 variables can lead to non-extended formulations that are ideal and smaller than the smallest extended counterpart. This allows such formulations to provide a significant computational advantage for disjunctive constraints related to the modeling of piecewise-linear functions. In this talk we describe a systematic geometric procedure to construct such non-extended formulations with a flexible use of 0-1 variables in an attempt to explain and expand on the success of the formulations from prior work with S. Ahmed and G. L. Nemhauser. This procedure is based on an embedding of the disjunctive constraint into a higher dimensional space and leads to several theoretical questions concerning the complexity of unions of polyhedra and the mixed basic semi-algebraic representability of unions of convex basic semi-algebraic sets.

Speaker: **Francisco Zaragoza** ((UAM Azcapotzalco, Mexico)
Title: *Traveling Repairman Problem on a Line with Unit Time Windows*

Abstract: Let $G = (V, E)$ be a graph and $r \in V$. For each $e \in E$, let $\ell_e > 0$ be the length of e. For each $v \in V$, a time window $[a_v, b_v]$ is given. A repairman starts in vertex r at time $t = 0$ and moves trough the edges of G at unit speed. The Traveling Repairman Problem consists of finding a route for the repairman that maximizes the number of vertices visited during their time windows. This problem is known to be NP-hard even when G is a tree and each time window has unit length (Frederickson and Wittman, 2012) or when G is a path and time windows are arbitrary (Tsitsiklis, 1992). The complexity of the remaining case, that is when G is a path and all time windows are unitary, is still open. Much work has been done in this case: there are approximation algorithms with guarantees 8 and $4 + \epsilon$ in quadratic time (Bar-Yehuda, Even, Shahar, 2005) and 3 in quartic time (Frederickson and Wittman, 2012). The algorithm with guarantee 8 has been improved, to get a guarantee of 4 in quadratic time (López, Pérez, Urban, and Zaragoza, 2014). All these algorithms use dynamic programming as the main tool. We have improved the analysis of this algorithm to show that it guarantees less than 3. Our main tool was setting up a linear program that describes the possible outcomes of the dynamic program, and solving it to find the worst possible outcome. Joint work with Cynthia Rodriguez (University of Waterloo, Canada)